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# PROPAGATION OF A RADIATION PULSE WITH WAVELENGTH $\lambda$ = 10.6 μm IN AMPLIFYING MEDIA

V. A. Levin, V. V. Netesov, and A. M. Starik

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Investigations of radiation amplification processes in multistage $CO_2$ amplifiers have recently intensified. Information about both the amplitude and phase characteristics of the radiation being amplified is necessary here to obtain high-intensity laser radiation in the far zone and small beam divergence.

Heating of the gas occurs in propagation of a collimated beam through a $CO_2$–$N_2$ amplifying medium because of the induced transitions from the level 00°1 to the level 10°0 and the subsequent vibrational–translational relaxation of the vibration energy of the $CO_2$ molecules. Moreover, because of the inhomogeneity in the intensity distribution over the beam radius (there is usually a Gauss distribution), thermalization of the gas over the beam radius will also occur nonuniformly.

Subsequently, the inhomogeneous temperature field will result in a nonuniform change in the refractive index over the beam radius, and will defocus the beam being propagated through the amplifying medium.

The influence of these processes on the amplitude–phase characteristics of the radiation being amplified was first investigated in [1]. However, questions of the influence of the possible inhomogeneity of the medium (particularly the inhomogeneity in the gain over the beam radius), and also of certain features of the vibrational-energy transfer processes on the radiation characteristics were not considered here.

On the basis of numerical modeling, propagation of a pulse of electromagnetic radiation with $\lambda$ = 10.6 μm through a $CO_2$–$N_2$–He amplifying medium with both a uniform and a nonuniform parameter field in the beam channel is examined here. Singularities in the production of the active medium are not taken into account here, and influence of the motion of the medium was assumed inessential.

The analysis was performed for pulses whose duration $\tau_I$ is much greater than the rotational–translational ($\tau_{R-T}$), vibrational–vibrational ($\tau_{V-V}$) relaxation times, and greater than the characteristic hydrodynamic time of the problem $\tau_p$ (the time of acoustic vibrations propagation across the beam) but less than the free convection, molecular diffusion, and heat conduction times. In this case, assumptions about the presence of a local Boltzmann distribution in each separate type of $CO_2$ and $N_2$ molecular vibration and on the existence of thermodynamic equilibrium between the translational and rotational motion of the mixture molecules are valid [2].

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Neglecting the vibrational and translational–rotational energy fluxes because of the presence of a translational and vibrational temperature gradient along the beam radius, the system of equations describing the change of state of the medium during pulse amplification can be represented in the form [2-4]

$$\rho C_p \frac{\partial T}{\partial t} = k_\nu I - \frac{R}{\mu}\sum_{i=1}^{2}\gamma_i \sum_{j=1}^{4}\Theta_{ij}\frac{d\varepsilon_{ij}}{dt}; \tag{1}$$

$$\frac{d\varepsilon_1}{dt} = \frac{p}{KT}\left[\frac{y_3 - y_1y_2\exp\left(-\frac{\Theta_3-\Theta_1-\Theta_2}{T}\right)}{(1-y_3)(1-y_1)(1-y_2)}\sum_{i=1}^{3}W_{312}^i\gamma_i + \frac{y_4 - y_1y_2\exp\left(-\frac{\Theta_4-\Theta_1-\Theta_2}{T}\right)}{(1-y_4)(1-y_1)(1-y_2)}W_{42}\gamma_2 + \right. \tag{2}$$

$$\left. + \frac{k_\nu I\mu^2}{\rho^2 N_A^2 h\nu_0\gamma_1} - \frac{y_1 - y_2^2\exp\left(-\frac{\Theta_1-2\Theta_2}{T}\right)}{(1-y_1)(1-y_2)^2}\sum_{i=1}^{3}W_{12}^i\gamma_i\right];$$

$$\frac{d\varepsilon_2}{dt} = \frac{p}{KT}\left[\frac{y_3 - y_1y_2\exp\left(-\frac{\Theta_3-\Theta_1-\Theta_2}{T}\right)}{(1-y_3)(1-y_1)(1-y_2)}\sum_{i=1}^{3}W_{312}^i\gamma_i + 3\frac{y_3 - y_2^3\exp\left(-\frac{\Theta_3-3\Theta_2}{T}\right)}{(1-y_3)(1-y_2)^3}\sum_{i=1}^{3}W_{32}^i\gamma_i - \right. \tag{3}$$

$$- 2\frac{y_2 - y_{20}}{1-y_2}\sum_{i=1}^{3}W_{20}^i\gamma_i + \frac{y_4 - y_1y_2\exp\left(-\frac{\Theta_4-\Theta_1-\Theta_2}{T}\right)}{(1-y_4)(1-y_1)(1-y_2)}W_{412}\gamma_2 +$$

$$\left. + 3\frac{y_4 - y_2^3\exp\left(-\frac{\Theta_4-3\Theta_2}{T}\right)}{(1-y_4)(1-y_2)^3}W_{42}\gamma_2 + 2\frac{y_1 - y_2^2\exp\left(-\frac{\Theta_1-2\Theta_2}{T}\right)}{(1-y_1)(1-y_2)^2}\sum_{i=1}^{3}W_{12}^i\gamma_i\right];$$

$$\frac{d\varepsilon_3}{dt} = \frac{p}{KT}\left[-\frac{y_3 - y_4\exp\left(-\frac{\Theta_3-\Theta_4}{T}\right)}{(1-y_3)(1-y_4)}W_{34}\gamma_2 - \frac{\mu^2 k_\nu I}{\rho^2 N_A^2 h\nu_0\gamma_1} - \frac{y_3 - y_1y_2\exp\left(-\frac{\Theta_3-\Theta_1-\Theta_2}{T}\right)}{(1-y_3)(1-y_1)(1-y_2)}\sum_{i=1}^{3}W_{312}^i\gamma_i - \right. \tag{4}$$

$$\left. - \frac{y_3 - y_2^3\exp\left(-\frac{\Theta_3-3\Theta_2}{T}\right)}{(1-y_3)(1-y_2)^3}\sum_{i=1}^{3}W_{32}^i\gamma_i\right];$$

$$\frac{d\varepsilon_4}{dt} = \frac{p}{KT}\left[\frac{y_3 - y_4\exp\left(-\frac{\Theta_3-\Theta_4}{T}\right)}{(1-y_3)(1-y_4)}W_{34}\gamma_1 - \frac{y_4 - y_{40}}{1-y_4}\sum_{i=1}^{3}W_{40}^i\gamma_i - \right.$$

$$\left. - \frac{y_4 - y_2^3\exp\left(-\frac{\Theta_4-3\Theta_2}{T}\right)}{(1-y_4)(1-y_2)^3}W_{42}\gamma_1 - \frac{y_4 - y_1y_2\exp\left(-\frac{\Theta_4-\Theta_1-\Theta_2}{T}\right)}{(1-y_4)(1-y_1)(1-y_2)}W_{412}\gamma_1\right]; \tag{5}$$

$$C_p = \frac{R}{\mu}\left(\frac{5}{2} + \sum_{i=1}^{2}\gamma_i\right), \quad \varepsilon_j = \frac{r_jy_j}{1-y_j}, \quad p = \frac{\rho RT}{\mu}.$$

Here $y_j = \exp(-\Theta_j/T_j)$; $\Theta_j$ and $T_j$ are, respectively, the characteristic and local vibrational temperatures of the j-th mode (the subscripts j = 1, 2, 3, 4 denote the symmetric, deformational, and asymmetric modes of $CO_2$ vibration and the molecular nitrogen vibration), $y_{j0} = y_j(T)$, $\rho$, T, p are the gas density, temperature, and pressure, R is the universal gas constant, K is Boltzmann's constant, $N_A$ is Avogadro's number, h is Planck's constant, $\mu$ is the mixture molecular weight, $\gamma_i$ is the molar fraction of the i-th component in the mixture (i = 1 is $CO_2$, i = 2 is $N_2$, and i = 3 is He; I is the intensity of the radiation acting on the gas, $k_\nu$ is the gain coefficient, $\nu_0$ is the transition frequency corresponding to the center of the line P20[00°1 → 10°0] of the $CO_2$ molecule, $W_{pk}$ is the rate constant of the elementary V–V' exchange and V–T relaxation processes (the subscripts p = k = 1, 2, 3, 4 correspond to the symmetric, deformational, and asymmetric types of $CO_2$ vibrations and the $N_2$ vibrations, while k = 0 corresponds to the unexcited state of any molecule). The temperature dependences of the rate constants for the elementary processes $W_{pk}(T)$ were selected in conformity with the recommendations [2, 5, 6]. It should be noted that the total probability of $CO_2$ asymmetric vibration energy relaxation over the channel $00^01 \rightarrow [03^10, 11^10] - W_{3\Sigma}^1$ [5], which is related to $W_{312}^1$ and $W_{32}^1$ by the relationship [3, 6]

$$W^i_{3\Sigma} = W^i_{32} + W^i_{312}\frac{1-y_2}{1+y_2} \tag{6}$$

is ordinarily determined experimentally. For a known function $\varphi_i = W^i_{312}/W^i_{32}$ the equation (6) permits evaluation of $W^1_{312}$ and $W^1_{32}$:

$$W^i_{32} = W^i_{3\Sigma}\left(\frac{1-y_{20}}{1+y_{20}}\varphi_i + 1\right), \quad W^i_{312} = W^i_{32}\varphi_i.$$

The value of $\varphi_i$ (i = 1, 2, 3) was here calculated from the data in [7]. An analogous procedure was also used in the determination of $W_{412}$ and $W_{42}$:

$$W_{42} = W_{4\Sigma}\left(\frac{1-y_{20}}{1+y_{20}}\varphi_4 + 1\right), \quad W_{412} = W_{4\Sigma}\varphi_4.$$

The gain coefficient $k_\nu$ at the center of the spectrum line P20[00°1 → 10°0] was calculated from the relationship [5]

$$k_\nu = \frac{\lambda_0^2}{8\pi^{3/2}}\sqrt{\frac{2RT}{\mu_1}}A_{mn}\frac{(2j''-1)\rho N_A\gamma_1}{\mu Z_r Z_V}H(\omega, 0)\delta N,$$
$$\delta N = y_3 B' \exp\left[-\frac{B'j''(j''-1)}{KT}\right] - y_1 B'' \exp\left[-\frac{B''j''(j''+1)}{KT}\right],$$

where $\lambda_0$ is the wavelength at the center of the amplifying transition line ($\lambda_0$ = 10.589 μm), B" and B' are the rotational constants of the levels 10°0 and 00°1, respectively, j" is the rotational quantum number of the lower level (j"= 20), H(ω, 0) is the Voight function, $Z_r$, $Z_V$ are the rotational and vibrational statistical sums, and the Einstein coefficient $A_{mn}$ is assumed equal to 0.187 $sec^{-1}$ [5].

Propagation of electromagnetic radiation along the derived direction (Oz) in a certain nonmagnetic medium with dielectric permittivity ε is described in a quasioptical approximation by the equation [8, 9]

$$2ik\left(\frac{\partial}{\partial z} + \frac{n_0}{c}\frac{\partial}{\partial t}\right)A + \Delta_\perp A + k^2\left(\frac{\varepsilon}{n_0^2} - 1\right)A = 0, \quad k = \frac{2\pi}{\lambda_0}n_0, \; \Delta_\perp = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right), \tag{7}$$

where A = A(r, z, t) is the complex electromagnetic wave amplitude, r is the distance from the beam axis in a plane perpendicular to the propagation direction, and n is the refractive index (the subscript 0 refers to the unperturbed medium). The electromagnetic wave intensity is related to the complex amplitude by the relationship $I = (cn/8\pi)|A|^2$, where c is the speed of light in a vacuum [10].

The complex dielectric permittivity ε of a medium is ordinarily written in the form

$$\varepsilon = (n + i(\lambda_0/4\pi)k_\nu)^2. \tag{8}$$

The change in the state of the medium during passage of resonance radiation results in a change in both the imaginary ($k_\nu$) and the real (n) parts of the dielectric permittivity. For $\tau_I \gg \tau_p$ the isobaric approximation ($p = p_0$) is valid, and a change in the refractive index is determined by the formula

$$\delta n = -(\delta T/T_0)(n_0 - 1). \tag{9}$$

The system of equations (1)-(9) is closed and permits determination of the amplitude–phase characteristics of the radiation with the self-action effect taken into account. We will examine the amplification of beams with a plane phase front and a Gauss transverse amplitude distribution

$$A(r, z = 0, t) = A_0 \exp(-r^2/2a^2)g(t), \tag{10}$$

where $A_0$ is the maximal amplitude, $a$ is the characteristic dimension of the beam and g(t) is a function characterizing the time modulation of the amplitude. The following boundary and initial conditions

$$A(r=\infty, z, t)=0, \quad \frac{\partial}{\partial r}A(r=0, z, t)=0,$$
$$T(r, z, t=0)=T_0, \quad \varepsilon_{ij}(r, z, t=0)=\varepsilon_{ij}(T_0), \quad T(r=\infty, z, t)=T_0, \tag{11}$$
$$\frac{\partial}{\partial r}T(r=0, z, t)=0$$

should here be satisfied. The system (1)-(9) with the boundary and initial conditions (10) and (11) was solved numerically. A fundamental difficulty in the numerical integration of the system (1)-(9) is the necessity to evaluate high-frequency phase oscillations and amplitude changes for the electromagnetic wave which occur even during beam propagation in a homogeneous medium. A method was proposed in [11] for the conversion of the propagation equation (7) by using the exact analytic solution for Gaussian beams being propagated in media with a linear coefficient of absorption. It permits limiting oneself to a calculation of changes in the electromagnetic wave amplitude and phase due just to nonlinear effects. This method, which is called "nonadaptive," is used even in this paper. Absolutely stable symmetric implicit difference schemes of second order accuracy [12] were used for the difference approximation of the differential equations. The solution of the system of difference equations at each step of the integration with respect to time was by an iteration method. In each iteration the difference analogs of the propagation (7) and energy (1) equations were solved by the right factorization method, and the kinetic equations (2)-(5) by the Newton method [12]. The computations were checked by satisfaction of the energy conservation law.

A specific analysis was performed for a step pulse $g(t) = 1$ for $0 \leq t \leq \tau_I$ with intensity $I_0 = 1$ kW/cm$^2$. It is considered that the length of the active zone amplifying the radiation is $L = 2$ m, while the characteristic beam dimension is $a = 0.1$ cm. The media $CO_2$–$N_2$ with $\gamma_1 = 0.1$ and $\gamma_2 = 0.9$ and $CO_2$–$N_2$–He with $\gamma_1 = 0.1$, $\gamma_2 = 0.4$, $\gamma_3 = 0.5$ were examined with the following unperturbed parameters: $T_1 = 2000°K$, $T_1 \equiv T_2 = T_3 = 400°K$, $T_3 = \dfrac{\Theta_3}{\Theta_4/T_4 + \dfrac{\Theta_3-\Theta_4}{T_0}}$, $p_0 = 10^{-2}$ MPa. Let us note that such parameters are typical for electrical discharge systems. We first examine the influence of the He content in the $CO_2$–$N_2$ mixture on the output pulse characteristics for a homogeneous parameter distribution in the medium.

As is known, the presence of He in a $CO_2$–$H_2$ mixture results in acceleration of the V–T relaxation of deformation vibrations of the $CO_2$ molecule, and consequently, in more intense depopulation of the lower level ($10^0$0) of the amplifying transition. In this case, saturation of the amplifying transition will occur at a later time than in a $CO_2$–$N_2$ gas. Hence, the energetic characteristics of the signal being amplified should be higher with He present in the mixture. Computations performed confirmed the assumption mentioned.

Represented in Fig. 1 for a pulse with $\tau_I = 90$ μsec are distributions of the relative amplitude $\bar{A} = A/A(0, z)$ over the beam radius at the amplifier output ($z = 2$ m) for the mixtures $CO_2$–$N_2$ (dashed lines) and $CO_2$–$N_2$–He (solid lines) at different times $t = 0$, 90 μsec (curves 1 and 2, respectively). Here and henceforth, the dimensionless beam radius $r' = r/a\sqrt{D}$ is given along the abscissa, where $a\sqrt{D}$ is the beam width ($D = (z/ka^2)^2 + (1 - z/f)^2$ and f is the radius of phase front curvature). It is seen that at all times in the $CO_2$–$N_2$–He mixture the amplitude of the signal being amplified in any section of the beam is higher than in the $CO_2$–$N_2$ mixture.

It should be noted that the $CO_2$–$N_2$ medium possesses a large energy reserve here. However, rapid saturation of the amplifying transition in the $CO_2$–$N_2$ mixture does not permit realization of this energy, as is easily seen from a comparison of the distributions of the vibrational $T_i$ and translational T temperatures and the gain coefficient $k_\nu$ (curves 1-6, respectively) along the beam radius for the two media under consideration represented in Fig. 2 (the distributions for $CO_2$–$N_2$ are given by dashes, and for $CO_2$–$N_2$–He by solid lines). It should be noted that inhomogeneous heating of the medium results in a certain blurring of the ray (thermal defocusing); however, this blurring is small for a pulse duration of $\tau_I = 90$ sec.

The singularities considered in the change in state of the $CO_2$–$N_2$ and $CO_2$–$N_2$–He media during passage of a radiation pulse result in the fact that the total energy in the ray emerging from the amplifier is greater at all times for the $CO_2$–$N_2$–He mixture than for the $CO_2$–$N_2$ mixture.

A situation when the gain coefficient profile is inhomogeneous is often encountered in real apparatus. Thus, upon exciting vibrations of the working gas molecules in an electrical

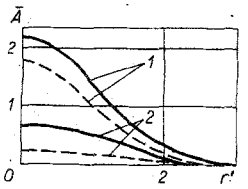


Fig. 1

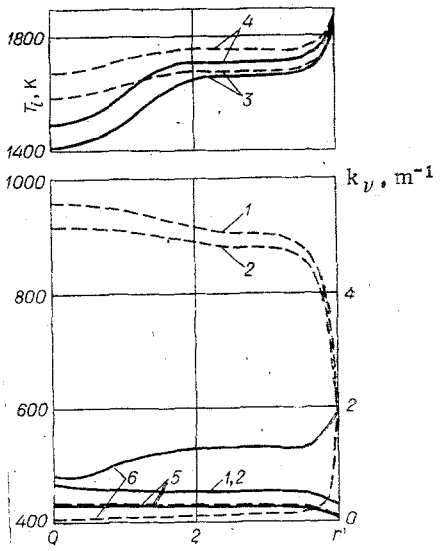


Fig. 2

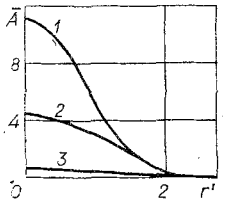


Fig. 3

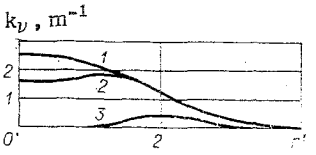


Fig. 4

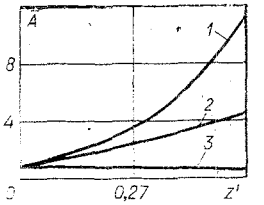


Fig. 5

discharge, the pumping intensity diminishes to the electrode circumference. Consequently, it is interesting to consider the influence of such an inhomogeneity on radiation pulse propagation and on the output characteristics of such an amplifier. A specific analysis was performed in an example of the $CO_2$—$N_2$—He = 0.1:0.4:0.5 medium with a symmetric gain coefficient profile relative to the beam axis (Gaussian). The distribution of the nonequilibrium parameters (vibrational temperatures $T_i$) relative to the beam axis was given as follows $T_1(r, z) = T_2(r, z) = T_1(0, z_0 + [400 - T_1(0, z)]F$, $T_4(r, z) = T_4(0, z) + [2000 - T_4(0, z)]F$, $T_2(r, z) = \Theta_3 T_4(r, z)T_0/[\Theta_4 T_0 + (\Theta_3 - \Theta_4)T_4(r, z)]$, where $F = \exp(-r^2/8a^2)$, $T_1(0, z) = T_2(0, z) = 400$ °K, $T_4(0, z) = 2000$ °K.

The gas pressure and temperature were assumed homogeneous, $p_0 = 10^{-2}$ MPa and $T_0 = 300$°K. The length of the amplifying medium along the ray was assumed to equal 2 m, while the effective beam radius is $a = 0.25$ cm and $I_0 = 1$ kW/cm$^2$. Shown in Fig. 3 is the distribution of the relative amplitude $\bar{A}/A(0, z)$ over the beam radius (r') at the output from the amplifier at different times t = 0, 2.6, 350 μsec (curves 1-3). It is seen that for t = 350 μsec magnification of the beam energy does not occur. Moreover, the amplitude of the signal being amplified is the same at time t = 0 for r' = 2 as at t = 2.6 μsec. Such behavior of the dependence $\bar{A} = f(r')$ is explained by singularities in the change in the gain coefficient over the beam radius in the presence of radiation. These singularities are illustrated in Fig. 4, in which the distribution $k_\nu = f(r')$ is represented for the middle section (z = 1 m) for t = 0, 2.6, 350 μsec. It is seen that for r' = 2 the gain coefficient is identical at t = 0 and 2.6 μsec.

Rapid saturation of the amplifying transition on the beam axis and a slower diminution in $k_\nu$ at its periphery results in the fact that the amplitude of the signal being amplified for $t > 0.5$ μsec at the beam axis $r' = 2$ becomes greater than the amplitude on the beam axis. An inhomogeneous gain coefficient profile also results in an inhomogeneous change in the translational temperature along the radius. This inhomogeneity increases with time. The refractive index here also changes analogously, resulting in thermal blurring of the ray, especially for large $\tau_I$ (greater than in a medium with homogeneous initial parameters). The effects noted are illustrated in Fig. 5 in which the change in $\bar{A}$ is shown along the track length $z' = z/k\alpha^2 D$ for $r' = 0$ at different times $t = 0$, 2.6, 350 μsec (curves 1-3). It is seen that at $t = 350$ μsec the amplitude of the signal being amplified diminishes as $z'$ increases.

Therefore, the computational analysis performed showed that for amplification of "long" pulses ($\tau_I >> \tau_p$), "thermal blurring" of the ray being amplified is possible even on moderate tracks. Thermal defocusing of the beam can be attenuated by diminishing the pulse duration.

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